Blossoms_-_Averages_Still_Flawed_v6

[MUSIC PLAYING]

GARRISON KEILLOR: That's the news from Lake Wobegon, where are the women are strong, all the men are good looking, all the children are above average.

DAN LIVENGOOD: I love Garrison Keillor's radio show. Lake Wobegon, where all the children can be above average.

RHONDA JORDAN: What's funny, Dan?

DAN LIVENGOOD: What do you mean? All the children can't be above average. Can they?

RHONDA JORDAN: Let's ask the class.

DAN LIVENGOOD: OK.

RHONDA JORDAN: Hello, class. This is Dan Livengood, and my name is Rhonda Jordan. And today we're here to talk to you about why averages are still flawed.

So, why do you think this was put into the closing lines and Garrison Keillor's radio show? Why is it funny? Do you think it's possible or impossible for all of the children of Lake Wobegon, to be above average? Discuss this with your neighbor, and with your teacher, and report back to us in about five minutes.

RHONDA JORDAN: So you see, if it's a national average, then it is possible for all of the children of Lake Wobegon, to be above average.

DAN LIVENGOOD: That's really cool. I hadn't thought of it that way. Huh, Lake Wobegon, where all of the children can be above average. Cool.

I love fishing.

RHONDA JORDAN: Me too. It's so relaxing. So, Dan, how many fish have you caught today?

DAN LIVENGOOD: Three so far. I caught 6 yesterday. So my average over the two days is 4 and 1/2 fish. It's kind of funny thinking about catching 1/2 a fish.

RHONDA JORDAN: Yeah, that's something I always found strange about averages. You know, in some cases the average is not even a possible outcome of the original situation.

DAN LIVENGOOD: Yeah. I wonder how many other situations are like this. Hey class, can you come up with any ideas of when the average may not be a possible outcome of the original situation? Talk it over with your classmates and your teacher. And we'll see you back in the classroom when you're done.

So class, welcome back. Did you come up with some examples when the average may not be a possible outcome of the original situation?

RHONDA JORDAN: Oh, I did.

DAN LIVENGOOD: OK, what's that?

RHONDA JORDAN: Well, when I'm bored, I like to flip coins.

DAN LIVENGOOD: OK, that's a bit strange.

RHONDA JORDAN: Sure. But hear me out.

DAN LIVENGOOD: All right.

RHONDA JORDAN: So on my coin, there's a head side and a tail side. So I like to see how many heads I get in a certain number of coin flips.

DAN LIVENGOOD: OK, so for instance if you flip the coin twice you would get 0, 1, or 2 heads, right?

RHONDA JORDAN: Exactly.

DAN LIVENGOOD: But are those outcomes equally likely?

RHONDA JORDAN: That's a great question. The answer is no, and here's why. In the first coin flip, it's equally likely that I get a heads or a tails. Then, in the second time flip, it's again equally likely that I get a heads or a tails. Putting that together, you can see that there are 4 potential final outcomes, each of which is equally likely.

DAN LIVENGOOD: Sure, I see that. And I also see that in those 4 possible outcomes there's 1 that has 0 heads, 1 that has 2 heads, but 2 that have 1 head. So it's actually twice as likely that you will count 1 head in your 2 coin flips.

RHONDA JORDAN: Yeah. So, from this information, we can calculate the average. I would expect to have 0 heads 1/4 of the time, 2 heads 1/4 of the time, and 1 head 1/2 of the time. Multiplying that out, the average number of heads I would count over 2 coin flips is 1.

DAN LIVENGOOD: Hey, wait a minute. That's the outcome of 1 of our coin flips. We're looking for situations where the average is not one of the outcomes.

RHONDA JORDAN: I know. Which is exactly why we need to see what happens when we flip the coin 3 times.

DAN LIVENGOOD: OK.

RHONDA JORDAN: Let's go back to our graphic with the 2 coin flip and add a third flip. As you can see, there are a total of 8 possible outcomes, each of which is equally likely. So 1 out of 8 times you get 0 heads. 3 out of 8 times you get 1 head. 3 out of 8 times you get 2 heads. And 1 out of 8 times you get 3 heads.

DAN LIVENGOOD: Ah, and in this case the average is 1.5, which is not a possible outcome. Hmm. So is it a coincidence that the average is actually the number of coin flips divided by 2.

RHONDA JORDAN: No, not in this case. But we'll leave that for the students to discuss with their teacher. Besides, flipping coins can help us out with another flaw of averages.

DAN LIVENGOOD: Oh, what's that?

RHONDA JORDAN: It's called regression towards the mean which is the same thing as saying regression towards the average.

DAN LIVENGOOD: OK, so what's that?

RHONDA JORDAN: Well its-- you know, this one will be more fun to experience. Let's turn things over to the teacher so the students can have fun with an activity. Then we'll meet you back at a special location where we can better demonstrate this flaw of averages.

Great throw, Dan.

DAN LIVENGOOD: Thanks.

RHONDA JORDAN: Welcome back. So do you know why we're out here on the baseball field?

DAN LIVENGOOD: Well yeah. I thought we were talking about regression towards the mean.

RHONDA JORDAN: We are.

DAN LIVENGOOD: OK.

RHONDA JORDAN: One example of regression towards the mean is a series of events. Like the wins and losses of a baseball team. You know, when a team goes on a long winning or losing streak?

DAN LIVENGOOD: Yeah, the winning streaks are great. The losing streaks aren't much fun, though.

RHONDA JORDAN: Well, understood, understood. But if they've won 7 games in a row, do you expect them to win another 7 games?

DAN LIVENGOOD: That would be awesome.

RHONDA JORDAN: Sure, but often does that happen?

DAN LIVENGOOD: Now that you mention it, not very often.

RHONDA JORDAN: And that's what regression towards the mean is all about. After an extreme event, like a long winning or losing streak, you team is simply more likely to regress towards the average number of wins and losses than go on another streak.

DAN LIVENGOOD: So wait a minute. What's the flaw with the average?

RHONDA JORDAN: There's nothing wrong with the average in this case. Regression towards the mean is actually a flaw of interpreting averages. The main point for teams to remember is to not be disappointed when a winning streak ends. Falling back into the average streak is normal.

DAN LIVENGOOD: Hmm, don't get disappointed. Regression towards the mean is normal. Thanks, that helps a lot.

RHONDA JORDAN: Any time. That's what friends are for.

DAN LIVENGOOD: [LAUGHING].

RHONDA JORDAN: Uh oh, what did you just think of?

DAN LIVENGOOD: Friends.

RHONDA JORDAN: What about friends?

DAN LIVENGOOD: Well there's an interesting flaw of averages involving friends. Did you know, that on average your friends have more friends than you do?

RHONDA JORDAN: What? No way.

DAN LIVENGOOD: It's true.

RHONDA JORDAN: Prove it.

DAN LIVENGOOD: Well, glad too. It's a good thing we brought some friends along.

RHONDA JORDAN: Mom? What are you doing here?

MRS. JORDAN: Yes, I was talking with Dad and he mentioned that you needed some help with the Friendship Paradox. So I'm here to help.

DAN LIVENGOOD: Thanks Mrs. Jordan. And let me introduce to you both my friend, Zan.

BOTH: Hi, Zan.

ZAN: It's great to meet you guys. So Dan, why did you invite me here today.

DAN LIVENGOOD: Well, with everyone here, we can explain the Friendship Paradox to the class. So class, here are the two questions. Number 1, how many friends does each member of this group of people have on average? And then number 2, averaging over each person in this group, what is the average number of friends that each of her or his friends has?

RHONDA JORDAN: So is there really a difference in these questions?

ZAN: Yeah, the two questions sound almost the same. And is even more than one average anyway?

DAN LIVENGOOD: Well that's the flaw for this Friendship Paradox. So class, we're going to turn it over to your teacher. And you're going to calculate the averages for the scenario that we just presented to you. Then, we're going to an activity for you where you're going to break into groups and calculate averages for different scenarios.

Have fun with that. Enjoy this. And we'll see you when you get back.

Welcome back. Did you have fun with this activity?

RHONDA JORDAN: So, Dan, when would you ever use something like this?

DAN LIVENGOOD: They're actually some really interesting ways to use this flaw of averages. One of them involves looking at the spread of an illness, like the flu. But that's a bit complex to explain here. We'll let the students and the teacher discuss this on their own if they're interested.

Let's go back to fishing. That's much simpler.

ZAN: Well, not necessarily. Remember that fishing competition we had last summer?

DAN LIVENGOOD: That again? I still say that I won.

RHONDA JORDAN: Wait, you don't know who won?

ZAN: Well, I think I won. And he thinks he won.

RHONDA JORDAN: How did that happen?

ZAN: It's all because of another flaw of averages. You can't take the average of the averages.

RHONDA JORDAN: Say what?

ZAN: Well, you can take the average of the averages. But it doesn't mean when you think it means. Here's what happened. Last summer Dan and I each took 30 fishing trips. But not always together.

DAN LIVENGOOD: Sometimes we would fish at the lake, sometimes we would fish at the river.

ZAN: And we'd call it a success if we caught at least 1 fish.

RHONDA JORDAN: OK, I follow you so far.

ZAN: Let's look at a table. On average, Dan was successful in catching at least 1 fish in 89% of his trips to the lake, and 43% of his trips to the river.

DAN LIVENGOOD: Zan on the other hand was successful in 83% of her trips to the lake and 33% of her trips to the river, on average.

RHONDA JORDAN: This looks pretty straightforward to me. I see why Dan thought that he won.

DAN LIVENGOOD: It sure looked that way.

ZAN: Until you look at our average success rate for all the fishing trips.

RHONDA JORDAN: What? How is that even impossible?

DAN LIVENGOOD: I know! It's true, I

ZAN: Won.

RHONDA JORDAN: That's crazy! How did that happen?

ZAN: Well, we can't spoil the surprise that easily. Let's have the class try to figure it out. So, what do you think? Why can't we take the average of the averages? And how do the numbers work out so that I won? Why don't you spend some time with your teacher and your classmates? See if you can figure it out and we'll see you back here soon.

Welcome back. Did you come up with the following table?

RHONDA JORDAN: Wow, that's crazy. And really cool. Great job, Zan.

DAN LIVENGOOD: I still can't believe the flaws of averages made me think that I won when Zan actually won.

ZAN: Well, it just shows you actually have to be pretty careful when you're taking the average of averages.

RHONDA JORDAN: It's amazing how many flaws of averages there are in our lives. Like the idea of catching half a fish reminds us that the average may not be a possible outcome of the event.

DAN LIVENGOOD: Then there's regression towards the mean, which tells us that a sports team is expected to return to an average sequence of wins and losses after an extreme event, like a long winning or losing streak.

ZAN: Don't forget about the Friendship Paradox. You don't get caught using the wrong average to answer a tricky question.

RHONDA JORDAN: And last but not least--

TOGETHER: All the children can be above average.

MRS. JORDAN: Hey you three, let's go fishing.

ZAN: Awesome, let's do it.

GARRISON KEILLOR: Well, it's been a quiet week in Lake Wobegon, Minnesota, my hometown. Where the skies are warm--

RHONDA JORDAN: Hello, I'm Rhonda Jordan.

DAN LIVENGOOD: And I'm Dan Livengood.

RHONDA JORDAN: We want to thank you for choosing this BLOSSOMS module entitled Averages Still Flawed. The goal of this lesson, just like our Flaws of Averages lesson, is to illustrate a few pit falls that may occur when using averages.

As we said in our first module, we are not trying to say that averages are bad. In fact, they're quite useful. We simply want students to be aware of these pitfalls so that they are able to interpret and use averages appropriately.

We hope the opening Lake Wobegon quote about all the children are above average sounds absurd to your students initially. Reactions like, that's impossible are encouraged. What we had in mind for the break is for you to guide your students to discover that, in fact, it is possible for all of the students of Lake Wobegon to be above average.

Two key points, and our view, is that if the children are all from one town, like Lake Wobegon, then it is impossible for all of them to be above average. However, if the average is a national average, then it's in fact possible that all the children in Lake Wobegon are above average. Let the students explore some ideas as they may come up with other examples.

DAN LIVENGOOD: The main point of the second segment is really part of the closing question, that the average may not be a possible outcome of the original situation. Our suggestion for this break is to guide the students in a thought exercise to start, thinking of situations where this might occur. The one thing that we ask in the break after the segment is that you close by discussing a coin flip.

In this discussion, we'd ask that you explain the idea of a random event, which in this case is the coin flip, and then also the expected value in whatever terminology best suits your students. The main point is to discuss a single coin flip where the expected number of heads on any given flip is 1/2, which is not a possible outcome of the event, where you could have 0 or 1. This will help set the stage for our next taped segment.

RHONDA JORDAN: For the break after the third segment, we suggest that you have all of your students flip a coin 8 times and then count the number of heads flipped. You could first ask what students expect average to be. Then we would suggest you write out the possible outcomes from 0 to 8 on a chalkboard and have the students walk up, stand in the line reflecting the number of heads that each one of them flipped.

Ultimately, they're going to create a human distribution of the number of heads flipped by the class. And for advanced students, you can discuss topics like binomial distributions. And the probability of flipping a certain number of heads in that distribution could again be discussed here. For less advanced students, simply calculating the average of this particular instance could suffice. Noting that it may or may not be what's expected.

After calculating the average in this particular instance, and drawing the distribution on the chalkboard, we ask that you have the student that flipped the most number of heads stay at the board while the other students are seated. You can then ask the class what they think will happen if this student flips his or her coin another eight times. Will he or she repeat their previous performance of a large number of heads? Hopefully they will suspect that the student's next attempt will regress towards the average. And then after rerunning the coin flips show that he or she does indeed regress towards the average.

One challenging situation will be if he or she actually beats their first total. If this happens, the best way to deal with it is to probably just have them try it again.

DAN LIVENGOOD: The fourth segment, the one about the Friendship Paradox, is one of the more complex ones that we have. But also is a lot of fun, and very interesting. So for the first part of the video break, what we'd like you to have them do with the students is really help them understand the difference between those two questions that we present them. The following ideas are simply our suggestions. Feel free to experiment with other ways to help your students really understand this.

For us, the first thing we did was actually create a structure to help visualize the math that's happening and that's being asked in each question. We've put on the website a table on one of the PDFs where you can find this table. And you can go look at it there.

For the second way to think about it, is actually to approach this as if you're thinking like a scientist. A scientist in general often looks at the two different extremes. So the first extreme is if you have a group of students where one student is friends with everyone else. But all of the other students are not friends with anyone other than that one student. Another extreme is if everyone in the group is friends with everyone else.

Again, we talk about the details of this on the website. So check out the PDF there.

After discussing the extremes, if you have time, one of the activities that we thought was going to be fun for this, you could have the students create groups of friends, say about 4 or 6 people in each group. And to make it random, you want to make this not contingent upon the social dynamics in the class. You could use the following process to create the friendship graph.

So for each pairing of 2 people in this group, have them flip a coin. If it's heads, then they are friends in this example. If it's tails, then they're not friends in this example. After building a graph like ours, they can then fill out the table, again on the website. Or do whatever else they would prefer to help actually calculate the averages for these two questions, to see the difference of what happens.

One very important note, it is possible after those coin flips that a group of say 6 students will end up in 2 independent groups of 3 students. Sorry, you'll end up with 2 independent groups of friends with 3 students in each group. That's OK, the process will still work. The averages will still work out. And they should just use the numbers from the original group and not break into smaller groups.

For more advanced students, also on the PDF on the website, we've got some more detail about this calculation particularly focusing on how variance is what causes this Friendship Paradox to always be true. This is likely most appropriate for students in a statistics course. Last but not least, you may get some questions about where do you use this? We've also again got some links on the website with some stories and some articles about the example we use in the segment about tracking illnesses like the flu, vaccinating people in a group, and things like that.

RHONDA JORDAN: For the break after the fishing competition, we suggest that you ask the students whether they think this could even be possible. To ultimately see what happened, you may have to guide them to work on this problem via a structure that we present in the PDF that can be found online. We would also suggest guiding them by asking, so what do we know?

This includes not only the percentage success rates shown in this segment but also the 30 total trips for both Dan and Zan. With that information, the total number of successful trips can be calculated as 17 and 22 for Dan and Zan respectively. From here, it will likely be trial and error in order to find the right combination of the rest of the numbers.

So in the case of Dan, you see that he's a better fishermen overall. More specifically, he's a better fishermen at each location. But, for this particular competition, where the winner caught the most fish in both locations, we see that Zan was smarter to fish at the location where the fish were more easily caught.

This is an example of Simpson's Paradox. And so you can look up online additional examples to share with your class.

DAN LIVENGOOD: So along with the lesson topics that we've included in this video, we have some additional material on the web page, including a homework problem that looks at a few flaws of averages in a real world situation. The problem considers where to place wind turbines on an island if you want to have the best source of electricity. We encourage you to take a look at it and hopefully find useful.

RHONDA JORDAN: Thank you for choosing BLOSSOMS.

DAN LIVENGOOD: We had a great time developing this module, Averages Still Flawed. We're sorry that we don't have a song and dance for you in this one. But if you haven't seen our first module, Flaws of Averages, we encourage you to check that out. Thanks and good luck.

[MUSIC PLAYING]