# *Averages: Still Flawed*

## Regression Toward the Mean and the Binomial Distribution

Supplement for the break after the third segment

**Classroom Activity**

To visually "see" the regression toward the mean, our suggested activity for this break in the ***Averages: Still Flawed*** module is to have all of your students flip a coin 8 times, having each of them count the number of heads flipped. You could first ask what the students expect the average to be, which we show in more detail below. Then, we would suggest you write out the possible outcomes from 0 to 8 on a chalkboard and have the students walk up, stand in the line reflecting the number of heads each flipped, which ultimately creates a human distribution of the number of heads flipped by the class. For advanced students, you could discuss topics like binomial distributions and the probability of flipping a certain number of heads in that distribution, which is again discussed in more detail below. For less advanced students, simply calculating the average of this particular instance should suffice, noting that it may or may not be what was expected.

After calculating the average in this particular instance and drawing the distribution on the chalkboard, we ask that you have the student that flipped the most number of heads stay up at the chalkboard while the rest of the students sit down. You could then ask the class what they think will happen if this student flips her or his coin another 8 times. Will she or he repeat their previous performance of a large number of heads? Hopefully they will suspect that this student's next attempt will "regress toward the average", and then after re-running the coin flips show that she or he does indeed regress toward the mean. One challenging situation will be if the student actually beats her or his first total. If this happens, the best way to handle it will probably to have them try it again.

**The Binomial Distribution, for more advanced students**

The coin flip activity is an example of a **binomial distribution** for a discrete random variable. For complete details on this topic, we recommend searching online or in available textbooks for "binomial distribution", as others have written quite effectively on the topic. What we include here is simply a quick summary.

To start, there is a formula for calculating the probability of counting any particular value, call it *k*, of the number of heads after making *n* coin flips.

For any value *k* such that $0 \le k \le n$, the probability of counting *k* heads in *n* coin flips is

$P(k|n) = \binom{n}{k} p^k (1-p)^{n-k}$, which is equivalent to $\mathrm{P(k|n)} = \frac{\mathrm{n!}}{\mathrm{k!(n-k)!}} p^k (1-p)^{n-k}$

For the case of a coin flip, $p = ½$, which simplifies the equation to $\mathrm{P(k|n)} = \frac{\mathrm{n!}}{\mathrm{k!(n-k)!}} \left(\frac{1}{2}\right)^n$

Let's say you have a class of 32 students. If you ask them to each flip a coin 8 times, count the number of heads and then walk up to the chalkboard to create a human distribution, you would expect to have the following distribution of students:

| | |
|---|---|
| 0 heads counted: | 0.125 students |
| 1 head counted: | 1 student |
| 2 heads counted: | 3.5 students |
| 3 heads counted: | 7 students |
| 4 heads counted: | 8.75 students |
| 5 heads counted: | 7 students |
| 6 heads counted: | 3.5 students |
| 7 heads counted: | 1 student |
| 8 heads counted: | 0.125 students |

Clearly you won't have a perfect distribution, but hopefully your activity yields a distribution similar to this. From there, if we took the average across all values of *k*, similar to what we show in the video segment, we would calculate the expected value of *n* coin flips to be

$E[k] = np = n/2$.

This is ultimately why the average of an **even** number of coin flips yields an average that **is** a possible outcome of the original situation, whereas the average of an **odd** number of coin flips yields an average that **is not** a possible outcome of the original situation.