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Teacher's Guide for the BLOSSOMS Video Module:
"Gravity Assist or Stealing a Planet's Angular Momentum and Getting Away With It"
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Dear Classroom Teacher, *(video to the classroom teacher)*

Your mission is to send a spacecraft to the outer solar system on a limited budget of rocket fuel; and that's not all, the trip must take you less time than what your fuel-carrying capacity allows. As the present video module will show, this mission is not mission impossible, **not even with high school students!**. It turns out to be relatively easy to achieve, and also to understand, with basic Newtonian mechanics.

This video module will present Gravity Assist, or what is wrongly referred to as the gravitational slingshot. "Gravity assist" is a maneuver performed by interplanetary spacecraft to travel farther on less fuel. It is a classic exercise in Newtonian mechanics. The basics are covered in high school physics curricula: conservation of linear momentum (to explain rocket propulsion), angular momentum, Newton's law of universal gravitation, and energy conservation. These concepts will be combined to explain orbits in the solar system. Gravity assists turn out to be carefully choreographed jumps between orbits. Only a single topic needed is usually not covered in high school curricula: distances in the solar system. A quick introduction into distances in the solar system will be enough to put the problem of fuel constraints in interplanetary travel into perspective and also to later highlight the importance of gravity assists in solving this fundamental problem. In short, "Gravity Assist" will integrate many seemingly disparate parts of high school Newtonian mechanics to present and explain a real application.

"Gravity Assist" will touch upon many exciting topics: rocketry, interplanetary travel, the solar system, etc. The most striking aspect of it will be to show that these topics can be understood using only high school physics, hence demonstrating the importance of physics in serving humanity's thirst for space exploration. The proposed video module is i) a new way of looking at, and applying various concepts of Newtonian mechanics studied in high school physics, ii) extends the theory studied in the classroom, and iii) a real world application to a very real problem, that of exploring the solar system.

Your students should have a working knowledge of all the above concepts. The best time to show this module is at the end of the mechanics part. You may wish to show in your class first, though it is not essential, Walter's Lewin's Ice Skater's Delight: http://blossoms.mit.edu/video/lewin.html, and Pervez Hoodbhoy's The Mystery of Motion http://blossoms.mit.edu/video/hoodbhoy.html.

You can find below a duet-by-duet teacher's guide.

Duet-by-Duet Teacher's Guide:

1) Duet #1: Introduction. Mission. Distances in the Solar System.

(Scene: Screen fades and "Classroom Activity" comes on for the duration of the assigned classroom activity, ~7 minutes. Brainstorming. Material Needed: Pens and paper.)

i. Classroom Teacher: Group Work – Brainstorming

a) The classroom teacher will repeat the question. The classroom teacher will ask the students to divide themselves into groups of 3 or 4 and to think of a practical approach. The classroom teacher will have to direct this group brainstorming exercise in the right direction.
b) The classroom teacher emphasizes that simple scaling implies that if one where to send Apollo to Saturn then it would take it 4000*3days=12,000days=33 years!
c) The students will invariably reach the conclusion that they need a really big, big rocket. The classroom teacher is to let the students "fight" it among themselves on how big the rocket (classroom teacher urges student to scale the numbers mentioned in the segment) should be and how long is too long for about a couple of minutes before he/she points out if the rocket is too big then it won't be able to lift itself from the ground in the first place.
d) The classroom will stress that all launches have to take place from Earth's surface and give a hint that the most of the initial amount (few million kg) of fuel needed is for lift off, going into orbit, and then breaking free. What the spacecraft actually carries with it for all its trip of several billions of km is less than 5 tons!
e) The classroom teacher will end the discussion by telling the students that the answer to this problem is in their physics textbooks and one only needs to know how to think about the stuff in the right way. He/she will say to the class that the present video module will show them how to do just that.

Background info the classroom teacher might find useful to know and could use as needed during the brainstorming session:

Apollo averaged about (384,400km)/(3days)(24 hours/day)(3600 second/hour)=1483m/s=5338km/h. The spacecraft rockets, not the Saturn V, provided part of this speed, the rest was provided by the Moon's gravity.

The Saturn V was jettisoned at stages with 97% percent of the initial mass being fuel, and was used up in the first several minutes. The third and last stage (SIVB) flung the spacecraft to the Moon. The Moon's gravity accelerated the spacecraft half way through the trip thereby making the trip actually shorter.

The Apollo spacecraft contained about 30,000kg of fuel for maneuvering, including for coming back to Earth! It is the heaviest spacecraft ever sent beyond Earth's orbit. All the rest are less than a few tons (including fuel).

2) Duet #2: Linear Momentum. Conservation of Linear Momentum. The Rocket Equation

(Scene: Screen fades and "Classroom Activity" comes on for the duration of the assigned classroom activity, ~7 minutes. Material Needed: Pens and paper. "Rocket Science" Calculations.)

i. Classroom Teacher: Group work – Playing with the rocket equation.

a) The classroom teacher will take a couple of minutes for re-enforcement and absorption: Ask the students to narrate in their own words how a rocket works. You don't have to ask each and every student. Present this as a general class question and take answers from 2-3 students. Make sure to have the answers of different students in the following duets.
b) Now divide the students into groups of 3 or 4. Keep the same groups as in the previous duet, if you wish.
c) Present the following exercise. It is similar to the ones dealt with in the video segment.
d) The last stage of the Saturn V rocket that inserted the Apollo spacecraft into a trans-lunar trajectory to the Moon, was 127,000kg, of which 72,000kg were fuel. It was capable of delivering 1,000,000 N of thrust. Its burnout time was 335 seconds. Calculate the rate at which fuel was consumed. Calculate the speed of exhaust gases. Calculate the final speed of the rocket at burnout, assuming an initial speed of 7.8km/s.
e) Give the groups about 3 minutes for the exercise and then solve the problem on the board.
f) The rate at which fuel is consumed is 72,000kg/335s=215kg/s
g) The speed of the exhaust gases is 1,000,000N/215kg/s=4.7km/s
h) The final speed of the rocket is 7.8+4.7 ln (127/(127-72))=7.8+4.7 ln(127/55)=11.7km/s
i) Classroom teacher will then go over the calculation again slowly.
j) The classroom teacher will point out that about 11.2 km/s of the above will be used to break free of Earth's gravity leaving about 500km/s to travel with. The Moon's gravity took over half way through the trip making the AVERAGE speed higher, about 1400km/s (remind students of duet #1).
k) The classroom teacher says that even at this speed, it would take 33 years to reach Saturn, and if you build a larger fuel tank for the spacecraft then you will need more fuel to lift it from ground and that will put you in a vicious cycle.
l) The classroom teacher will then point out that the rockets used nowadays to send spacecraft to the outer solar system are ordinary as far as these things are concerned.
m) The rockets used to lift off spacecraft that eventually reach Jupiter, Saturn, and beyond are actually 1/3 of the Saturn V. So obviously, large rockets won't do the trick. They are very expensive. Saturn V, which is the most powerful rocket ever built, is not enough to send spacecraft to the outer solar system. Extra help is needed.
n) The classroom teacher will ask the students to guess the nature of the extra help.
o) The classroom teacher could give the students a hint about gravity and the trip to the Moon, but should be quick to point out that the planets in the outer solar system are very far and consequently their gravity by itself is too weak to pull on the spacecraft from the vicinity of Earth.

3) Duet #3: Gravity + Newton's 2nd Law = Orbits

(Scene: Screen fades and "Classroom Activity" comes on for the duration of the assigned classroom activity, ~5 minutes. Material Needed: Pens, paper, strings, soft masses (for safety reasons). "Orbits" Demos & Calculations.)

i. Classroom Teacher: Group work – Playing with Orbits: Demos & Calculations

a) The classroom teacher will take a couple of minutes for re-enforcement and absorption: Ask the students to narrate in their own words the three laws of Kepler . You don't have to ask each and every student. Present this as a general class question and take answers from 2-3 students. Make sure to have the answers of different students in the following duets.

b) Now divide the students into groups of 3 or 4. Keep the same groups as in the previous duets, if you wish.

c) Ask the students: How can a planet resist the Sun's gravity and not fall towards? Give the rotating mass attached to a string analogy. Ask the students to attach soft masses to the end of a string and rotate the mass above their heads at safe distances from each other. The string will become taut. The tension in the string (inwards) is balanced by the outward centrifugal force. The result is a mass going in a circular orbit. The tension in the string is sort of analogy to the gravitational force. The classroom teacher may opt to do this demo in front of the class if strings and/or soft masses or safe distances cannot be secured.

d) The classroom teacher will present an exercise on orbits: periods, radius, energy. The exercise will compare the orbital properties about of Venus and Neptune. The aim of this exercise is to help the students develop an intuition about orbits.

e) Background info. Venus: mass 4.87×10^{24} kg, period: 224.7 days, semi-major axis: 1.082×10^{8} km. Neptune: mass 1.02×10^{26} kg, period: 164.8 days, semi-major axis: 4.495×10^{9}km. Sun: mass 1.99×10^{30}kg (for the sake of comparison)

f) Assuming a perfectly circular orbit for Venus (radius ~ 1.085×10^{8}km) calculate the energy per mass for this orbit. It takes about 225days to complete one orbit (2 x pi x radius), to find speed. E/m= -0.5 G(Msun)/radius. Msun=1.99×10^{30} kg. G=6.67×10^{-11} $N.m^2.kg^{-2}$.

g) Do same thing for Neptune.

h) Classroom teacher says that the energy of Venus is more negative than energy for Neptune. Venus is closer to the Sun and hence is more tightly bound than Neptune. So to move away from the Sun you need to get your energy less negative, you need to increase your energy. Where can you get it from? How can you do that? Let's see the rest of the module!

i) As an optional aside, the classroom teacher could then ask the students what will happen if a satellite fires its rockets forward, and take votes. The classroom teacher will then explain what will happen: firing rockets forward will provide backwards force, the satellite will drop to a lower orbit and hence the speed there is higher. While in orbit, if you wish to go faster you need to fire forward, to oppose your motion!

4) Duet #4: Angular Momentum. Gravity Assist I

(Scene: Screen fades and "Classroom Activity" comes on for the duration of the assigned classroom activity, ~3 minutes. Material Needed: Pens, paper. Angular Momentum Calculations.)

i. Classroom Teacher: Group work – Angular Momentum: Calculations

a) The classroom teacher will take a couple of minutes for re-enforcement and absorption: Ask the students to narrate in their own words what angular momentum is. You don't have to ask each and every student. Present this as a general class question and take answers from 2-3 students. Make sure to have the answers of different students in the following duets.

b) Now divide the students into groups of 3 or 4. Keep the same groups as in the previous duets, if you wish.

c) Calculate the angular momentum of a 1000kg spacecraft on Earth (it has the same orbital parameters as Earth). Calculate the angular momentum of a 1000kg spacecraft on Halley comet. The classroom teacher provides the masses, speeds, and distances. Earth, at perihelion 0.983AU and 30.26km/s. Halley comet at perihelion 0.586AU and 54.44km/s.

d) L(@earth)=1000*30.26*1000m/s*0.983*150,000,000,000m=4.46 x 10^{18} kg m^2/s

e) L(@halley)=1000*54.44*1000m/s*0.586*150,000,000,000m=4.79 x 10^{18} kg m^2/s

f) The difference in angular momentum between the two orbits is: 0.33 x 10^{18} kg m^2/s

g) The classroom teacher can give the energies at the 2 orbits:

h) E(@earth)=-442 x 10^9J. E(@halley)= -25 x 10^9J. The energy spacecraft at Halley orbit is 417 x 10^9J higher than if it were at Earth's orbit, or in other words, still ON Earth.

i) The classroom teacher then points out that the orbit of Halley's comet is highly elliptical, taking it to beyond Pluto's orbit.

j) The classroom teacher says that to go to Saturn, for example, you would want to put your spacecraft on an elliptical orbit similar to Halley's comet, and once you arrive close to your destination, jump again off that orbit.

k) So where can you get that extra energy and angular momentum?

5) Duet #5: Gravity Assist II

(Scene: Screen fades and "Classroom Activity" comes on for the duration of the assigned classroom activity, ~5 minutes. Material Needed: marbles or balls, light sturdy boxes.)

i. Classroom Teacher: Hands-on Activity

a) The classroom teacher will take a couple of minutes for re-enforcement and absorption: Ask the students to narrate in their own words what happens during a collision. You don't have to ask each and every students. Present this as a general class question and take answers from 2-3 students. Make sure to have the answers of different students in the following duet.

b) Now divide the students into groups of 3 or 4. Keep the same groups as in the previous duets, if you wish.

c) Material needed (for each group of 3-4 students): one marble, or ping pong ball, or tennis ball, book or box, a flat surface (floor).

a) Have the students first bounce the marble off the stationary box and note what happened to the speed.

b) Have one student push the box on the floor, another send the marble on a head-on collision trajectory, and the others note what happened to the speed.

c) The classroom teacher will wrap up by pointing out that the marble stole linear momentum from the MOVING box: this is a "Collision Assist"; the collision assisted the marble to get a much higher speed. The linear momentum was transferred through a direct contact between the colliding masses.

6) Duet #6: Gravity Assist III

(Scene: Screen fades and "Classroom Activity" comes on for the duration of the assigned classroom activity, ~5 minutes. Material Needed: baseball+bat or racket + ball, or racket + birdie.)

i. Classroom Teacher: Hands-on Activity. Calculations.

a) The classroom teacher will take a couple of minutes for re-enforcement and absorption: Ask the students to narrate in their own words what is a gravity assist. You don't have to ask each and every student. Present this as a general class question and take answers from 2-3 students.

b) Now divide the students into groups of 3 or 4. Keep the same groups as in the previous duets, if you wish.

c) Material needed (for each group of 3-4 students): baseballs+bats, rackets + balls, and/or rackets+birdies. Pencils, paper.

d) Classroom teacher can ask the students to play with rackets and birdies to experiment with the angular momentum swings (as in the video segment).

e) The birdie picks up angular momentum from the racket. The classroom teacher points out here that the interaction between the birdie and the racket is repulsive, hence the birdie approaches the racket in the same direction as its trajectory. Classroom teacher requests from students to keep this in mind. A table tennis racket and ping pong ball will also work, same for tennis racket and ball.

f) Calculate the amount of angular momentum transferred when a spacecraft get a gravity from Jupiter. This is a back of the envelope calculation. Suppose that the spacecraft approaches Jupiter with at a speed of 10 km/s. Jupiter at the time would be making around 13km/s. If the angle of approach is about 50^0 with velocity vector of Jupiter, calculate the final velocity of the spacecraft. Assume that Jupiter, and the spacecraft at that time, is 750,000,000,000m away from the Sun. Calculate the initial and final momenta of a 1000kg spacecraft. Compare this to the angular momentum of Jupiter.

g) The classroom teacher compares the initial and final angular momenta to show that the change for the spacecraft is huge while for the assisting planet negligible.

h) The classroom then goes over the problem, pointing out the velocity gained and how that increased velocity takes the spacecraft to its destination in shorter time with less fuel. Most of the fuel on the launch pad is actually used for lift off.

i) Gravity assist usually give a few km/s. This might sound small. BUT, putting an intact Saturn V in space is prohibitive. That would require tens of Saturn Vs to launch to space. These few km/s are crucial. The spacecraft is getting them for free.

References:
http://phet.colorado.edu: A collection of superb applets (I used Collisions Lab, My Solar System, and Gravity and Orbits)

http://www.messenger-education.org/Interactives/ANIMATIONS/grav_assist/gravity_assist_menu.html Simulations of the gravity assist for the New Horizons spacecraft, and also information about the MESSENGER mission to Venus.

http://www2.jpl.nasa.gov/basics/grav/primer.php: A gravity assist primer by NASA.

http://saturn.jpl.nasa.gov/ : Website of the Cassini-Huygens mission. Look under "Mission Overview" for more information about the gravity assists done by this spacecraft and more information about the location and speed of the spacecraft. Please look at the Cassini Mission Overview Supplement in the "Quick Facts" page under "Mission Overview." This file contains info about the Cassini orbiter, launch vehicle, and speeds before and after the gravity assists.

http://pluto.jhuapl.edu/ : Website of the New Horizons mission to Pluto and the Kuiper Belt. I got the rockets videos and simulations from there.

http://www.esa.int/esaCP/SEMXLE0P4HD_index_0.html : The gravity assist webpage on the website of the European Space Agency (ESA).

I thank you for your patience and for using this module in your classroom. I hope that you enjoyed it! If you have any questions you can contact me at bsabra@ndu.edu.lb

Regards,

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