The Monty Hall Problem

Or How to Outsmart a Game-show and Win a Car

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The purpose of our lesson is to teach students to think about how to make decisions in the face of uncertainty about the outcomes. We use the Monty Hall problem as a fun example of an instance where you have to make a decision with multiple possible outcomes, each with a different probability. We teach students how to use a decision tree to determine the best choice for both the Monty Hall problem and other situations. We try to get students to think about other situations in their lives where they make decisions with limited knowledge about the outcomes, and whether or not "making the correct decision" depends on the eventual outcome. We also try to teach how limiting cases can be helpful for developing an intuition. By the end of how video, students should be able to make and use a decision tree and appreciate some of the everyday circumstances where they apply.

Decision trees are based upon probability, and students need to have a basic knowledge of probability before starting our lesson. This includes being able to calculate the probability of a given outcome out of a set, for example, rolling a six-sided dice and landing on the number 4. Students also need to be able to calculate the probability of two independent events both occurring, for example, rolling a four on a dice and then flipping a coin and getting tails. This is all that is needed to make and use a decision tree.

Over the course of the video, students complete the following exercises:

- Discussing with their neighbors about whether or not to switch doors
- Playing the 3 door Monty Hall game in pairs with three envelopes
- Constructing the decision tree for the Monty Hall Problem
- Playing the 26 envelope Monty Hall game
- Discussing with their neighbors about times when you make the optimal decision but don't get the optimal outcome

Here are some useful things to keep in mind as students work through the activities:

- Students may work in pairs or in groups, which ever you think is better for your class
- You may use anything you'd like for the prizes in the Monty Hall game. Small candy is a good choice. If you use something three dimensional, you will need something else similar in size and shape to include in the envelopes so students cannot tell what is in it just by looking.
- When playing the 3-door game in pairs, you may want to have all the students who won raise their hand, and then count subset that switched and

won. You can see how close it is to 2/3. If it is not 2/3, you may want to talk to your class about the role of sample size: how you need a large number of trials to get the theoretical value. You could even have your class play multiple rounds to see how the fraction of winners who switched gets closer to 2/3.

- When playing the 26-envelope version, select 3 students to participate: one host, one assistant, and one contestant. The envelopes should be labeled A-Z, so they can be identified. Ask the contestant to leave the room and have the host and assistant put the prize into one envelope. Have the contestant come back and select an envelope. Then have the assistant open up 24 envelopes before asking if the contestant wants to switch. In the rare case (1/26) that the contestant selects the envelope with the prize, at the end of the exercise you can ask your class whether he or she still made the right choice by switching. This serves as an excellent intro to that segment of the video.

The Monty Hall Problem has a rich history. You may want to consider sharing this with your students. You can learn about the history from the Wikipedia page about the problem. Your students may be interested to learn how this problem stumped many PhDs.

We hope you and your students enjoy this lesson and that it reflects the fun we had creating it. We welcome any feedback, suggestions, or comments you would like to offer. You may share them by emailing Sam at shames@mit.edu.

Thank for your choosing our lesson.