

## MIT BLOSSOMS

### The Towers of Hanoi Problem Teachers Guide Prepared by Dr. Richard Larson

This is a highly interactive, problem-focused BLOSSOMS module on an advanced math topic: **Recursion**. It has applications in many areas of math and computer science, and in everyday life. We have tried to make it accessible to any college-bound math class with a certain level of math maturity, say at the junior or senior levels in high school but even down to middle school. A knowledge of exponential growth is valuable, as is a knowledge of set theory, but neither is mandatory.

Each student should have a stack of disks in front of her in order to work with their own personal Towers of Hanoi. As stated in the video, this can be as simple as a stack of 5 coins or washers, stacked in ascending order, smallest at the top. But the class should have at least one version of the Towers of Hanoi with a hard surface foundation that can be passed from student to student.

#### **ACTIVITY 1 Gaining familiarity with the problem.**

Each student will try to solve  $N=3$  **Towers of Hanoi** problem. They discuss with neighbors and with you, their teacher. They record the number of moves. Some will successfully solve it. Those students should try to explain their strategies to others.

#### **ACTIVITY 2 Guess the Minimum Number of Moves.**

The goal here is very simple: To get the students to see exponential growth. It's a bit tricky, as the growth goes as  $(2^N) - 1$ , not strictly as  $2^N$ . Teacher guides discussion towards this goal.

#### **ACTIVITY 3 Exponential Growth and Patterns.**

Students discuss patterns and see that Small is moved every other time. Large is moved only once, to its final destination. Medium is moved twice, in some sort of pattern that is emerging. Students discuss how this pattern may evolve for larger values of  $N$ .

#### **ACTIVITY 4 Beginning of Recursion.**

Students try to see how to use this recursive reasoning to solve the  $N=4$  problem. Some will be successful, some will not. The successful ones should explain their reasoning to the rest of the class.

#### **ACTIVITY 5 Guest Appearance, Showing How Easy Recursion is.**

See Video Teacher's Guide for detailed instructions on this class activity.

#### **ACTIVITY 6 Students Volunteer to Serve as Drs. 3, 4 and 5.**

You ask for three volunteers. It may be that the class is so into this, that teams are set up, each with three players, and they compete to see which team can finish first.

#### **ACTIVITY 7 Formal Definition of Recursion. Optional Stop Point**

If time is running short or if you do not want to go into the technical details of recursion and proof by induction, you may choose to STOP here. Whether you do or not, you may want to have a conversation with the class about the definition of recursion and ask them how they could be actors in computing  $(10!)$  by recursion.

**ACTIVITY 8 Formalizing Recursion in an Equation.**

This should be a quick break. The students need to show their understanding by writing down the recursive equation for Sol(5,1,3). Neighbors can help!

**ACTIVITY 9 Proving Exponential Growth.**

This may be the first time students see **Proof by Induction**. So you may want to expand a conversation on it. But once they believe the result, the Monks' problem is a fun exercise. Some calculators give up on the large numbers involved. And computing the exact number of seconds is not so important. You may suggest to the students that they can approximate  $2^{10}$  as 1,000. This helps in getting to the answer. And, of course they need to figure out how many seconds in a year.

**ACTIVITY 10 The END.**

There is no Activity 10. But we imagine that there could be lots of excitement and strategies discussed in class. We provide the even/odd rule but do not give the students a cookbook recipe for solving the general problem. We deliberately leave the remaining details to them and their practice with the puzzle.