The Art of Approximation in Science and Engineering:
How to Whip Out Answers Quickly

By Stephen Hou
Ph.D. Candidate, Department of Electrical Engineering and Computer Science
MIT

The purpose of my video is to show students how to think more freely about math and science problems. I examine some concepts in ways that students might not have seen in school. In particular, scientific calculations can be tedious and long. Many textbooks and teachers emphasize getting exactly the right answer. But sometimes, getting an approximate answer in a much shorter period of time is well worth the time saved. In this video, I explore techniques for making quick, back-of-the-envelope approximations that are not only surprisingly accurate, but are also illuminating for building intuition in understanding science. This video is intended as a supplement to formal math and science education.

This video touches upon 10th-grade-level Algebra I and first-year high-school physics, but the concepts covered (velocity, distance, mass, etc) are basic enough that science-oriented younger students would understand. Knowing more advanced physics concepts such as Newton's laws, force, and momentum are not necessary. In fact, I think it would be wonderful for students who have not yet been formally introduced to physics to think about the exercises I pose here because it will encourage them to be more creative in formulating solutions.

There are five exercises that students are asked to think about in this video.

(1) What is 29 times 31? Don't use a calculator or the standard procedure for multiplying numbers on paper, which is tedious!

(2) Estimate 3/17 as a percentage without using long division.

(3) Say we have a free-swinging pendulum. Think about what characteristics of the objects involved or their environment will determine the period of one round-trip swing. Come up with an equation for the period in terms of these characteristics. If you're having trouble getting started, you can assume that the period (in units of s) depends only on the length of the pendulum L (m) and the gravitational acceleration g (m/s^2).

(4) How close can you get to a black hole without being sucked in? Again, think about what physical parameters you need, either about the universe (like the role g played in the pendulum example) or about the object in question (like the role length played in that example). Here is a hint: Use
* G (gravitational constant), units of m^3/(kg s^2)
* M (mass of black hole), units of kg
* c (speed of light), units of m/s

During the breaks, I suggest that you encourage students to work together in small groups of two or three. My examples are sufficiently unconventional that I think the students will have a lot of fun discussing them among themselves rather than pondering them alone. When you feel that the mini-conversations have gotten to the point where many students are stuck or have ideas about how to move forward, it would be an appropriate time to get back to the video.

If you have time in your classroom, I also suggest asking students to present their solutions for their classmates. For more advanced or motivated students, you may even ask them to invent their own approximation problems as homework assignments and challenge their classmates to try to solve them.

Dr. Sanjoy Mahajan, the Associate Director of the MIT Teaching and Learning Laboratory, has many materials, including a draft of his textbook, on approximation in math and science available on his website:

http://www.inference.phy.cam.ac.uk/sanjoy/

I suggest that you show these materials to your students, perhaps even before showing this video module.

One final note: In segment four of this video module you will see some calculations on the board behind me that do not relate to anything we have discussed. Don't worry! These calculations were part of an exercise that was edited out of the video.

I had a lot of fun creating this module, and I hope you and your students enjoy it as well. Good luck!