Teacher's Guide:

The overall objective of the lesson will be to introduce the students to a branch of mathematics that is not usually covered in a high school curriculum – graph theory. Graph theory has many applications, from airplane routing to circuit chip production and hopefully as a result will be very appealing to the students.

An introduction to graph theory does not require high level mathematics because we will be building the students' knowledge from the ground up. The prerequisites for this lesson plan are arithmetic and understanding of what is a set. In the lesson I will describe use sets to refer to nodes and edges of our graphs. If students aren't familiar with sets, they should still be able to follow the lesson.

This lesson is composed of two separate sections. The first is based on the Seven Bridges of Konigsberg and involves finding an Eulerian tour through the city. The second is based on the Chinese Postman Problem and involves finding the shortest path around the graph. The second portion builds of the first, and can be covered in a separate second class session.

Most aspects of the lesson are relatively straight forward, the more challenging parts are parts 3 and part 6. To ensure that the students understand the points made in part 3 you may have to repeat the same points as in the video. For part 6 the complexity is in keeping track of all the points. It may be helpful to lay out the following four points:
1. The sum of all the degrees in the graph is always even.
2. The number of all the even degree nodes' is even
3. The sum of all the odd nodes' degrees is always even.
4. The number of odd degree nodes is even.
After laying out this general structure, you can go point by point and show how they follow one another.

One of the great things about this topic is that each student can take some time to try and come up with their own paths for the graphs and for additional practice you can easily come up with your own graph, of the appropriate level of complexity, to either really challenge your students or just solidify the material. One way to challenge the students is to ask them to solve a Chinese Postman problem with 6 odd nodes. At that point the number of possible matchings will make the problem challenging. At the same time, I wouldn't recommend graphs with more than 6 odd nodes.

You can also challenge the students to think of some areas in their lives where using Graph Theory could be useful. For example, you can talk about how you can find the shortest path from one node to another in a graph, using the Dijkstra algorithm. You can even encourage that students do a project on an interesting application of something in Graph Theory.